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Tax Evasion and Growth:

a Banking Approach

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Tax Evasion and Growth: a Banking Approach

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Abstract:

The paper formalizes the relation between flat taxes and growth when there is a competitive equilibrium tax evasion. A decentralized tax evasion service is supplied by the banking sector. The bank production function follows the financial intermediation microfoundation approach, with deposits as an input. Across a class of endogenous growth models, tax evasion decreases the effective tax rate, and thereby lessens the negative effect of taxes on growth. And as the tax rate rises, tax evasion causes the growth rate to fall by less. Underlying the results is a fiscal principle whereby tax evasion creates, or magnifies, a rising demand price sensitivity to higher tax rates.

JEL: E13, E62, H26, O41

Keywords:

Tax evasion, financial intermediation, endogenous growth, and flat taxes.

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1 Introduction

Tax evasion is a worldwide phenomena.[1] It alters the effective tax rate, and this can change the growth effect of taxes, such as that found in Stokey and Rebelo (1995). And across the whole range of tax rates, tax evasion can cause an entirely different profile of the effect of taxes on growth.

The profile of the effect of taxes on growth is not well-known. Will decreasing tax rates have the same marginal effect on growth no matter what the level of tax rates? This can be important, for example since US tax rates have varied significantly, with the top marginal personal income tax rate falling from 92% in 1952 down to 35% in 2008, and with top bracket corporate tax rates falling over the same period from 52% to 35%. And this postwar downward tax rate trend has been seen internationally to a significant degree, such as with low flat tax rate regimes arising across "New" Europe and Russia. Endogenous growth theory predicts that such significant decreases in the average tax rates should have significant effects on growth in the long run; but does the growth effect differ according to the level from which taxes are decreased?

Using the flat-tax approach of Stokey and Rebelo (1995) as an abstraction, this paper shows in standard endogenous growth economies that the effect of flat taxes on growth can be linear, or almost linear, with similar marginal effects on the balanced path growth rate, of either capital or labor tax changes, regardless of the level of the taxes. Such near linearity may not be realistic. It would imply that as tax rates rise, growth rates fall steadily and even become negative at some point. In contrast, the tax-growth profile evidence for the inflation tax implies that the growth rate decreases by increasingly less as the inflation tax rises(Gillman, Harris, and Matyas 2004).

The inflation tax literature shows that including a means of avoiding the inflation tax, through credit that is produced in a banking sector, changes the profile of the effect of the tax on growth from a nearly linear one to a more nonlinear one (Gillman and Kejak 2005). Applying this same banking

[1] One associated measure is the size of the underground economy, on which Schneider and Enste (2000) has focused.

approach, this paper allows for a competitive equilibrium evasion (rather than avoidance) of capital, labor, and value-added goods taxes, again through a banking sector.[2] This produces a significantly non-linear profile of the effect of taxes on growth that would appear to be more plausible than the profile without evasion.

In allowing for evasion, the paper also presents a principle of public finance and growth. The principle is simple: the higher the tax rate, the greater the substitution towards evasion activity, and the higher is the price elasticity of demand for declared income. As the tax rate is increasingly evaded, its burden on the economic growth rate is increasingly less. Conversely, as tax rates are lowered and the taxes are increasingly less evaded, then the marginal positive effect of the tax reduction on growth is increasingly bigger. This may be good growth news for the tax reduction trend seen internationally.

Here, as with inflation tax avoidance, we model tax evasion so that it makes the effective tax rate less than the statutory rate. As the tax rate increases, the effective tax rate increases; but because of evasion the effective tax rate increases by an increasingly lesser amount. Since the growth rate falls as the effective tax rate increases, the result is that the growth rate falls by a smaller amount as the tax rises. The lesser fall in the growth rate from tax rate increases, and its explanation in terms of the rising price elasticity of demand for the taxed good, is a plausible principle of fiscal finance that holds with tax evasion or avoidance within classes of endogenous growth economies.

The paper first sets out the simplest economy that exhibits the principle at work. In a physical-capital-only, Ak economy, a decentralized banking sector is set up that provides a tax evasion service that enables the representative agent to report only a fraction of capital income to the government tax authority, while laundering the remaining of the capital income back into regular income through the banking system. The result is that an increase in the tax rate on capital income causes the growth rate to change in proportional to the fraction of income that the consumer reports to the government.

[2] Banking is the typical vehicle for tax evasion in practice. For example, the bank UBS was reported to be complicit in provide tax evasion services for clients (Mollenkamp, Simpson, and Frangos 2008). Lichtenstein banks were recently reported to be a conduit for international tax evasion (Dougherty 2008).

Because this fraction of reported income declines as the tax rate increases, the marginal decrease in growth becomes smaller as the tax rate increases. At the same time, the price elasticity of demand of the consumer for the reported income increases.

The model is then extended to have both human and physical capital. The effect of a capital tax change on growth in this case now comprises the tax evasion term of the Ak model, plus the standard Stokey and Rebelo (1995) tax effect on the return to capital. Next the tax evasion result on growth is shown for the labor tax. An Ah economy is presented with only human capital, with a resulting lesser magnitude of the tax's growth rate effect. This is rather more complicated than for the capital tax because the labor tax works on growth through its effect on leisure, and on the return to human capital, rather than directly taxing the return to physical capital. Both the labor and capital taxes with evasion are then both set within the full economy with human and physical capital, and a consumption tax (VAT) is also added. Similar results of the effect of all three taxes on growth with evasion are presented, along with comparison to the inflation tax, as based on an increasing price sensitivity to the tax.

2 The Ak Economy with Tax Evasion

The representative consumer owns the goods producer, who has a production function that is linear in only physical capital, and the consumer invests in capital and earns capital income by renting it to the goods producer. There is also a separate bank sector owned by the consumer that produces the tax evasion service that enables the consumer to report only a fraction of the capital rental income received from the goods firm.

The representative consumer utility is a function only of consumption goods, c_t; assuming a log form, and given an initial capital stock k_0, the consumer's total continuous time, discounted, utility stream is

$$V(k_0) = \int_0^\infty (\ln c_t)\, e^{-\rho t} dt. \tag{1}$$

Physical capital k_t is allocated between goods production and banking production, with the share of capital going to goods denoted by s_{Gt}, and to banking production by s_{kt}; the shares add to one:

$$s_{Gt} + s_{kt} = 1. \tag{2}$$

Investment i_t of capital, with a depreciation rate of δ_k, allows capital to be accumulated over time:

$$\dot{k}_t = i_t - \delta_K k_t, \tag{3}$$

and is simply allocated out of goods output y_t without additional adjustment costs, so that

$$\dot{k}_t = y_t - c_t - \delta_K k_t. \tag{4}$$

With the real interest rate denoted by r_t, the agent earns capital income from the two sectors equal to $r_t k_t \left(s_{Gt} + s_{kt}\right)$, which is the same as $r_t k_t$. However there is a tax on capital income at the rate of τ_k, on income that is reported to the government. To avoid paying taxes on some fraction of the capital income, with this fraction denoted by $a_{kt} \in [0, 1]$, the agent pays for the tax evasion service at a competitively determined market price. This price, denoted by p_{kt}, is per unit of tax evasion services, denoted by κ_t.

As a Leontieff one-to-one technology is assumed in the bank sector between the quantity of tax evasion services and the amount of income that is "laundered" by the bank (and so evades taxes), the quantity demanded of tax evasion services will be equal to the quantity demand of unreported, or "undeclared," income. [3] This means that the total income on which taxes are paid is equal to $a_{kt} r_t k_t$; net of tax income this is $\left(1 - \tau_k\right) a_{kt} r_t k_t$. The value of the rest of the income, that is laundered by the bank for a fee so that

[3] The demand for tax evasion services can be framed more formally within a Becker (1965) household production function. Specifically, let the consumer produce the consumption of goods by combining inputs of the purchased goods, and the income necessary to purchase the goods. This is a Beckerian Leontieff technology whereby the value of the goods and the amount of the income are in a one-to-one ratio. Second, the income itself is also household produced through the combination of either of two perfect substitutes: reported income or unreported income. The production of the unreported income is the result of combining the quantity of tax evasion banking services with the same quantity of real income, in a Leontieff one-to-one fashion. This set-up is parallel to that of Gillman and Kejak (2005) for credit services.

no taxes are paid on this fraction of income, is equal to $\kappa_{kt} = (1 - a_{kt})\, r_t k_t$; net of the tax evasion fee this income equals $(1 - p_{kt})\,(1 - a_{kt})\, r_t k_t$.

The consumer also receives income from a government transfer, denoted by v_t (equal to tax proceeds) and from a dividend due to ownership of the bank. The dividend comes because the consumer buys a share in the bank with each dollar deposited in the bank, as with mutual bank charters. The price of each share is fixed at one. There is no capital gain, but instead the profits are distributed in proportion to the deposits (or shares) held, so that after distribution of the dividends the bank has zero remaining profit. Denote the dividend rate per deposit as r_{kt}, and the quantity of deposits as d_{kt}. Then the total dividend income is $r_{kt} d_{kt}$. This makes the consumer's budget constraint, written in terms of equation (4), as

$$\begin{aligned} \dot{k}_t &= (1 - \tau_k)\, a_{kt} r_t k_t + (1 - p_{kt})\,(1 - a_{kt})\, r_t k_t + r_{kt} d_{kt} \\ &\quad - c_t + v_t - \delta_K k_t. \end{aligned} \tag{5}$$

The consumer deposits all capital income in the bank, so that

$$r_t k_t = d_{kt}. \tag{6}$$

2.1 Goods Producer Problem

Production of goods output, y_t, uses a linear technology in capital, $s_{Gt} k_t$,

$$y_t = A_G s_{Gt} k_t, \tag{7}$$

with $A_G > 0$. The firm, takes the prices of capital services, r_t, as given, and maximizes profit by choosing capital inputs

$$\max_{\{s_{Gt} k_t\}} \Pi_{Gt} = A_G s_{Gt} k_t - r_t s_{Gt} k_t. \tag{8}$$

This implies in equilibrium the fixed interest rate of

$$r_t = A_G. \tag{9}$$

2.2 Bank Production Problem

Taking the price of tax evasion services p_{kt}, the rental price of capital r_t and its production function as given, the bank maximizes its profit in a competitive fashion. Profit Π_{kt} is defined as the total revenue, being the tax evasion fee p_{kt} times the quantity of tax-evaded ("laundered") dollars that are produced, where this quantity is denoted by κ_t; costs are the rental costs $r_t s_{kt} k$ and the residual dividend payouts on deposits, $r_{kt} d_{kt}$. The profit maximization problem is

$$\max_{\{s_{kt}k_t, d_{kt}\}} \Pi_{kt} = p_{kt}\kappa_{kt} - r_t s_{kt} k_t - r_{kt} d_{kt}; \tag{10}$$

subject to its production technology of the tax evasion service. The tax evasion production function is assumed to be a CRS technology in capital, and deposited funds (financial capital) as in the financial intermediation microeconomic industry literature of Sealey and Lindley (1977), Clark (1984) and Hancock (1985):[4]

$$\kappa_{kt} = A_k \left(s_{kt} k_t\right)^{\omega_k} \left(d_t\right)^{1-\omega_k}, \tag{11}$$

where $\omega_k \in (0,1)$. The resulting equilibrium condition is that the cost of capital equals its marginal product;

$$r_t = p_{kt}\omega_k A_k \left(\frac{s_{kt}k_t}{d_t}\right)^{\omega_k - 1}. \tag{12}$$

This gives a solution for the capital to deposits input ratio $\frac{s_{kt}k_t}{d_t}$ as

$$\frac{s_{kt}k_t}{d_{kt}} = \left(\frac{r_t}{p_{kt}\omega_k A_k}\right)^{\frac{1}{\omega_k - 1}}$$

Substituting the equilibrium input ratio into the production function, and dividing by d_t, yields the ratio of tax evasion dollars to the deposits:

$$\frac{\kappa_{kt}}{d_{kt}} = A_k \left(\frac{\omega_k A_k p_{kt}}{r_t}\right)^{\frac{\omega_k}{1-\omega_k}}. \tag{13}$$

[4] Clark (1984) assumes that financial intermediary services are produced with a CRS function of labor, capital, and deposited funds; here we are postulating an economy without labor and so this factor is omitted, while maintaining the CRS assumption in the two factors of capital and deposits.

The amount of dollars that evade taxes is given as the fraction $1 - a_{kt}$ of capital income, or $(1 - a_{kt}) r_t k_t$. And this evasion service is what the bank is producing, so that

$$\kappa_{kt} = (1 - a_{kt}) r_t k_t. \tag{14}$$

And since the total deposits d_{kt} is equal to the capital income $r_t k_t$, it follows from equations (13) and (14) that

$$1 - a_{kt} = A_k \left(\frac{\omega_k A_k p_{kt}}{r_t} \right)^{\frac{\omega_k}{1-\omega_k}}; \tag{15}$$

this gives the supply of the unreported income as a function of the price of the tax evasion service p_{kt}. It results from equating the marginal benefit of producing the tax evasion to the marginal cost; or from equation (12),

$$p_{kt} = \frac{r_t}{\omega_k A_k \left(\frac{s_{kt} k_t}{d_t} \right)^{\omega_k - 1}}, \tag{16}$$

and the price of the service p_{kt} equals the marginal cost of the tax evasion output, which is the marginal factor price r_t divided by the marginal factor product, $\omega_k A_k \left(\frac{s_{kt} k_t}{d_t} \right)^{\omega_k - 1}$.

The CRS property of the production function (11), or the first-order condition with respect to d_{kt}, implies that the Cobb-Douglas coefficient $1-\omega_k$ is equal to the factor income $r_{kt} d_{kt}$ divided by the total revenue, in this case $p_{kt} \kappa_{kt}$. Substituting in for κ_{kt} from equation (14), it follows that

$$1 - \omega_k = \frac{r_{kt} d_{kt}}{p_{kt} (1 - a_{kt}) r_t k_t}. \tag{17}$$

From equation (6), the deposits d_t are equal to $r_t k_t$; with equation (17) this implies that

$$r_{kt} = p_{kt} (1 - \omega_k) (1 - a_{kt}). \tag{18}$$

The dividend rate r_{kt} is a fraction $(1 - \omega_k) (1 - a_{kt})$ of the tax evasion price p_{kt}, and the fraction rises as does the supply of the tax-evaded income.

2.3 Government

The government receives tax revenues only on reported capital incomes and pays a lump sum transfer of v_t; making the government budget constraint:

$$a_{kt}\tau_k r_t k_t = v_t. \tag{19}$$

2.4 Equilibrium

A competitive equilibrium for this economy consists of a set of allocations $\{c_t, a_{kt}, k_t, s_{Gt}, s_{kt}, d_{kt}\}$, a set of prices $\{r_t, p_{kt}, r_{kt}\}$, the government's policy $\{\tau_k, v_t\}$, and the initial condition k_0 such that

1. given the price of capital services, r_t, the banking fees, p_{kt}, and the returns to deposits, r_{kt}, the consumer maximizes utility $V(k_0)$ in equation (1) with respect to $\mathbf{u_t} \equiv (c_t, a_{kt}, d_{kt})$ and subject to its budget constraint (5), and to the deposit constraint (6);

2. given the price of capital, r_t, the goods producing firm maximizes profit Π_{Gt} in (8), with respect to its input of capital $s_{Gt}k_t$;

3. given the price of capital, r_t, the return to deposits, r_{kt}, and the fee for credit services, p_{kt}, the bank maximizes its profit Π_{kt} in (10) with respect to its input of capital $s_{kt}k_t$ and its input of deposits d_{kt};

4. the government budget (19) is always satisfied;

5. and all markets clear at the given prices.

The equilibrium margins from the first-order conditions are standard except for the effect of the tax evasion, which the next proposition highlights. Here, the existence of an interior competitive equilibrium, in which both the tax on reported income and the fee on tax evasion services are paid, implies the solution for the equilibrium price of the tax evasion service.

Proposition 1. The competitive equilibrium price of tax evasion services for capital income tax evasion is equal to the tax rate;

$$p_{kt} = \tau_k. \tag{20}$$

Proof. This follows directly from the consumer's first-order condition for fraction of reported income, a_{kt} (not shown).

The proposition, based as far back in the literature as Becker (1968), says that the consumer will spend on the margin an amount equal to the cost of paying (not evading) the tax, which is the tax rate τ_k. Thus the supply of the fraction of unreported income in equation (15) is a positive function of the level of the tax rate;

$$1 - a_{kt} = A_k \left(\frac{\tau_k \omega_k A_k}{r_t} \right)^{\frac{\omega_k}{1-\omega_k}}. \tag{21}$$

Tax evasion output $1 - a_{kt}$ increases as the tax rate increases. Solving equation (21) for τ_k, and using that $r_t = A_G$ from equation (9), Figure 1 graphs the convex ($\omega_k > 0.5$) upward sloping supply and the demand at $\tau_k = 0.3$ for unreported income; with $\omega_k = 0.3$, $A_k = 0.6$, and $A_G = 0.08$.

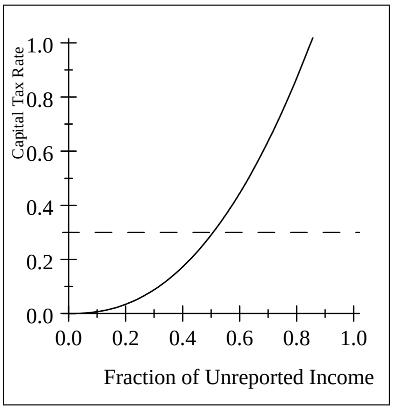

Figure 1: The Supply of Unreported Income $1-a_{kt}$; and Demand at $\tau_k = 0.3$.

The demand for unreported income (dashed line) is perfectly elastic at the price of τ_k. However, since the unreported income and reported income are perfect substitutes to the consumer, when finding income to purchase goods, the supply of unreported income in equation (21) also implies the demand for reported income a_{kt}, which falls as the tax rate increases. Key to the market for tax evasion is that it creates an effective tax rate that is less than the actual tax rate, through the return to the consumer of the dividend r_{kt} from the bank supplying the tax evasion service.

Corollary 1. The effective tax rate equals $\tau_k - r_k$.

Proof. The consumer's equilibrium conditions imply that the return on capital, net of taxes and the payment of tax evasion fees, is equal to $r_t \left[(1-\tau_k) a_{kt} + (1-p_{kt}) (1-a_{kt}) + r_{kt} \right]$, where r_{kt} is given by equation (18). The amount $\tau_k a_{kt} r_t$ is paid in taxes per unit of k_t; the amount $p_{kt} (1-a_{kt}) r_t$ is per unit the cost of tax evasion, and this cost is reduced by the dividends on the deposits in the evasion bank, paid at the rate of r_{kt} per dividend. This makes the effective tax rate equal to $\tau_k a_{kt} + p_{kt} (1-a_{kt}) - r_{kt}$. By Proposition 1, this effective rate reduces to $\tau_k - r_{kt}$.

The effective tax rate $\tau_k - r_{kt}$ can also be written as $\tau_k a_{kt} + \tau_k \omega_k (1-a_{kt})$, which can be thought as a weighted average of the average cost of the tax when reporting the income and when not reporting the income, with the weights a_{kt} and $1-a_{kt}$; the weighted average falls as a_{kt} decreases and more income is unreported.[5] The lower effective tax as a result of tax evasion affects the growth rate along the balanced-growth path, as the next proposition states.

2.5 Balanced Growth Rate

Proposition 2. Along the balanced growth path, the growth rate is given by

$$g = r (1-\tau_k + r_{kt}) - \delta_K - \rho, \tag{22}$$

where $r_{kt} = \tau_{kt} (1-\omega_k) (1-a_{kt}) \leq \tau_k$, and

$$\frac{\partial g}{\partial \tau_k} = -A_G a_{kt}, \tag{23}$$

$$\frac{\partial g^2}{\partial^2 \tau_k} = -A_G \frac{\partial a_{kt}}{\partial \tau_k} > 0 \tag{24}$$

where a_{kt} is given by equation (21).

Proof. By equation (18) and Proposition 1,

$$r_{kt} = \tau_{kt} (1-\omega_k) (1-a_{kt}), \tag{25}$$

[5] The average (before dividend) cost of the tax evasion when not reporting the income is given by $\frac{r_t s_{kt} k_t}{\kappa_t} = \tau_k \omega_k$, which is less than the average cost of τ_k when reporting income.

where $a_{kt} = A_k \left(\frac{\tau_k \omega_k A_k}{r_t}\right)^{\frac{\omega_k}{1-\omega_k}}$ by equation (21). Substituting into equation (22) for r_{kt}, using equation (25), gives that

$$g = r_t \left[1 - \tau_k + \tau_{kt} \left(1 - \omega_k\right) \left(1 - a_{kt}\right)\right] - \delta_K - \rho; \tag{26}$$

further substituting into equation (26) for $1 - a_{kt}$, and using equation (9) to substitute for r_t, gives the growth rate in terms of only exogenous parameters:

$$g = A_G \left[1 - \tau_k \left(1 - (1 - \omega_k) \left[A_k \left(\frac{\tau_k \omega_k A_k}{r_t}\right)^{\frac{\omega_k}{1-\omega_k}}\right]\right)\right] - \delta_K - \rho; \tag{27}$$

taking the derivative, gives that $\frac{\partial g}{\partial \tau_k} = -A_G a_{kt}$. Then it follows from equation (21) that $\frac{\partial g^2}{\partial^2 \tau_k} = -A_G \frac{\partial a_{kt}}{\partial \tau_k} > 0$.

Corollary 2. With no tax evasion, when $A_k = 0$, the tax linearly affects the growth rate.

Proof. With $A_k = 0$, by equation (21), $a_{kt} = 1$, and by Proposition 2, $\frac{\partial g}{\partial \tau_k} = -A_G$.

The preceding proposition and corollary show first that the dividend return r_{kt} makes the effective tax $(\tau_k - r_{kt})$ less than the statutory rate τ_k. Then it is shown that the growth rate falls with the tax rate τ_k increase, in proportion to the fraction of reported income a_{kt}. And the growth rate falls by marginally less, as the tax rate increases and the fraction of reported income falls. So the growth rate falls by a decreasing amount in a nonlinear fashion, as compared to a linear decrease when there is no production of the tax evasion service. This is illustrated in Figure 2, with the solid line showing the nonlinear case with tax evasion, and the dashed line showing the standard linear case with no tax evasion.[6]

And it follows that the more productive is the bank sector in producing tax evasion, the smaller is the decline in the growth rate (in Figure 2, an increase in bank productivity pivots up the solid line):

Corollary 3. An increase in bank sector productivity factor A_k causes a decrease in the magnitude of the growth effect for any given tax rate τ_k.

[6] Figure 2 assumes the parameters of $\omega_k = 0.3$, $A_G = 0.08$, $\delta_k + \rho = 0.04$, and $A_k = 0.6$.

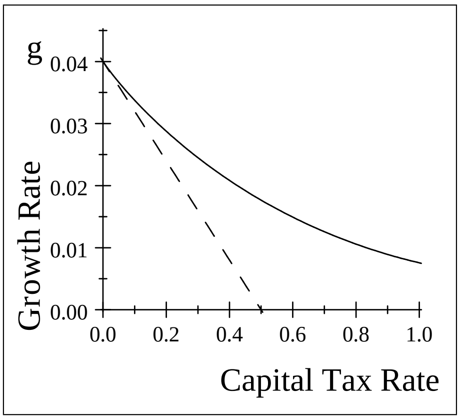

Figure 2: The Effect on Growth of the Capital Tax τ_k

Proof. From Proposition 2 and equation (21),
$\frac{\partial g}{\partial \tau_k} = -A_G a_{kt} = -A_G \left[1 - A_k \left(\frac{\tau_k \omega_k A_k}{r_t}\right)^{\frac{\omega_k}{1-\omega_k}}\right]$. Therefore
$\frac{\partial g^2}{\partial A_k \partial \tau_k} = A_G \left(\frac{\tau_k \omega_k}{r_t}\right)^{\frac{\omega_k}{1-\omega_k}} \left(\frac{1}{1-\omega_k}\right) A_k^{\frac{1}{1-\omega_k}-1} > 0.$

The link between the effective tax , growth, and the price elasticity of demand of reported income results because of the dependence of the change in the effective tax rate on the share of reported income a_{kt}. Intuitively, a tax rate increase causes a marginally smaller growth rate decrease because of an increasing price elasticity of demand for reporting income relative to the tax rate. Formally, consider the effect of the tax rate on the price elasticity, and on the elasticity of substitution between the reported and unreported income, in the following proposition and corollary:

Proposition 3. The price elasticity of demand for reported income with respect to the tax rate rises in magnitude as the tax rate rises.

Proof. This is seen by defining the price elasticity as

$$\eta_{\tau_k}^{a_{kt}} \equiv \frac{\frac{\partial a_{kt}}{\partial \tau_k}}{\frac{a_{kt}}{\tau_k}},$$

and computing it using equation (21) as

$$\eta_{\tau_k}^{a_{kt}} = -\frac{1 - a_{kt}}{a_{kt}} \frac{\omega_k}{1 - \omega_k}. \tag{28}$$

From equation (15) and Proposition 1, $\frac{\partial a_{kt}}{\partial \tau_k} < 0$, and so the price elasticity $\eta_{\tau_k}^{a_{kt}}$ rises in magnitude as the tax rate τ_k rises.

Corollary 4. The elasticity of substitution between reported income and unreported income rises in magnitude as the tax rate rises.

Proof. Define the elasticity of substitution as $\varepsilon = \frac{\partial\left(\frac{a_{kt}}{1-a_{kt}}\right)}{\partial\left[\frac{\tau_k}{\left(\frac{r_t}{\omega_k(A_k)^{1/\omega}}\right)}\right]} \frac{\left[\frac{\tau_k}{\left(\frac{r_t}{\omega_k(A_k)^{1/\omega}}\right)}\right]}{\frac{a_{kt}}{1-a_{kt}}}$, so that the relative price between reported income and unreported income is $\frac{\tau_k}{\left(\frac{r_t}{\omega_k(A_k)^{1/\omega}}\right)}$. Then it follows from equation (21) that

$$\varepsilon = -\frac{\omega_k}{1-\omega_k}\frac{1}{a_{kt}}. \tag{29}$$

From equation (28), and equation (29) of Proposition 2, it is clear that

$$\varepsilon = (1-a_{kt})\,\eta_{\tau_k}^{a_{kt}}, \tag{30}$$

and that $\frac{\partial \varepsilon}{\partial \tau_k} < 0$.

One of Marshall (1920) four laws of factor demands is that the elasticity of substitution between factors equals the share of the one factor, in this case $1-a_{kt}$, factored by the price elasticity of demand for the other factor, in this case $\eta_{\tau_k}^{a_{kt}}$. Thus the tax effects on the price elasticity of demand, and the elasticity of substitution, are both driven by the tax effect on the fraction of reported income a_{kt}. And therefore the result that the growth rate effect of the tax is smaller in magnitude as the tax rate rises, is synonymous with the result that the price elasticity of demand for reported income rises in magnitude as the tax rate increases. These effects are equally synonymous with a rising elasticity of substitution between the reported and unreported income.

3 The Capital Tax with Physical and Human Capital

The economy is extended to postulate tax evasion of the capital tax but now within an endogenous growth economy with both human and physical capital, as in King and Rebelo (1990). It is shown that the tax evasion causes a similar effect on growth as was shown for the Ak economy.

The banking sector will be specified to use inputs of effective labor and deposits, instead of capital and deposits, a simplification from using all three inputs of labor, capital and deposits. Effective labor is also used in goods production and human capital investment; physical capital is an input in goods production and human capital production.

The representative agent utility function now depends upon both goods c_t and leisure x_t, as given by

$$u\left(c_t, x_t\right) = \ln c_t + \alpha \ln x_t \tag{31}$$

with $\alpha \in R_+$. One unit of time is allocated among working in goods production, l_{Gt}, in human capital investment, l_{Ht}, in the bank sector, l_{kt}, and as leisure x_t :

$$l_{Gt} + l_{Ht} + l_{kt} + x_t = 1. \tag{32}$$

The share of physical capital is allocated to goods production s_{Gt}, and to human capital production s_{Ht} :

$$s_{Gt} + s_{Ht} = 1. \tag{33}$$

Goods and banking production are decentralized sectors from which the agent earns labor and capital income. In order to avoid capital taxes, the agent reports again only a fraction a_{kt} of the earned capital income.

With h_t and k_t denoting the human and physical capital stocks at time t, the agent's accumulation of physical and human capital is given by

$$\dot{h}_t = i_{Ht} - \delta_H h_t, \tag{34}$$

$$\dot{k}_t = i_t - \delta_K k_t, \tag{35}$$

where

$$i_{Ht} = A_H \left(l_{Ht} h_t \right)^{\varepsilon} \left(s_{Ht} k_t \right)^{1-\varepsilon}, \tag{36}$$

while physical capital investment is simply allocated out of goods output y_t without additional adjustment costs:

$$i_t = y_t - c_t. \tag{37}$$

Since the goods and bank sectors are decentralized, and given the real wage and real capital rental rate of w_t and r_t, the agent receives income from goods and banking labor, $(l_{Gt} + l_{kt}) w_t h_t$, and capital income $s_{Gt} r_t k_t$. It is assumed again that there is only one tax in the economy, the proportional capital income tax τ_k. This makes the capital taxes that are paid by the agent equal to $(1 - \tau_k) a_{kt} s_{Gt} r_t k_t$.

The agent again pays a proportional fee p_{kt} to the bank for the tax evasion services. The quantity of tax evasion services that the agent buys, denoted by κ_{kt}, is equal to the dollars of unreported capital income, $(1 - a_{kt}) s_{Gt} r_t k_t$. Thus the unreported capital income received by the agent is equal to

$(1 - p_{kt}) (1 - a_{kt}) s_{Gt} r_t k_t$. And since the representative agent owns the bank, according to the deposits of capital income placed in the bank, there are also dividends paid to the agent equal to $d_{kt} r_{kt}$, where d_{kt} is again the quantity of deposited funds, and r_{kt} is the dividend yield per deposit.

The total income constraint can be written in terms of equations (35) and (37) as

$$\begin{aligned} \dot{k}_t &= (l_{Gt} + l_{kt}) w_t h_t + (1 - \tau_k) a_{kt} s_{Gt} r_t k_t + (1 - p_{kt}) (1 - a_{kt}) s_{Gt} r_t k_t \\ &\quad + r_{kt} d_{kt} - c_t + v_t - \delta_K k_t. \end{aligned} \tag{38}$$

An additional constraint specifies that the amount of bank deposits are equal to the capital income from the goods and banking sectors:

$$s_{Gt} r_t k_t - d_{kt} = 0. \tag{39}$$

3.1 Goods Producer Problem

Production of goods output, y_t, uses a CRS technology in capital, $s_{Gt}k_t$, and effective labor, $l_{Gt}h_t$;

$$y_t = A_G \left(l_{Gt}h_t\right)^{\beta} \left(s_{Gt}k_t\right)^{1-\beta}. \tag{40}$$

The firm, takes the prices of capital and labor services, r_t, and w_t, as given, and maximizes profit by choosing effective labor and capital inputs

$$\max_{\{l_{Gt}h_t, s_{Gt}k_t\}} \Pi_{Gt} = A_G \left(l_{Gt}h_t\right)^{\beta} \left(s_{Gt}k_t\right)^{1-\beta} - w_t l_{Gt}h_t - r_t s_{Gt}k_t. \tag{41}$$

First-order conditions imply that

$$w_t = \beta A_G \left(s_{Gt}k_{\ t}\right)^{1-\beta} \left(l_{Gt}h_t\right)^{\beta-1}, \tag{42}$$

$$r_t = (1-\beta) A_G \left(s_{Gt}k_{\ t}\right)^{-\beta} \left(l_{Gt}h_t\right)^{\beta}. \tag{43}$$

3.2 Bank Production Problem

Bank profit Π_{kt} is similar to equation (10), subject to a similar production function to equation (11). But now, to simplify presentation of the remaining section of the paper, only labor is used instead of capital as an input along with deposits. This gives that

$$\max_{\{l_{kt}h_t, d_{kt}\}} \Pi_{kt} = p_{kt}\kappa_{kt} - w_t l_{kt}h_t - r_{kt}d_{kt}; \tag{44}$$

subject to

$$\kappa_{kt} = A_k \left(l_{kt}h_t\right)^{\omega_k} \left(d_t\right)^{1-\omega_k}, \tag{45}$$

where $\omega_k \in (0,1)$, and in equilibrium

$$w_t = p_{kt}\omega_k A_k \left(\frac{l_{kt}h_t}{d_t}\right)^{\omega_k - 1}. \tag{46}$$

Then

$$\frac{l_{kt}h_t}{d_t} = \left(\frac{w_t}{p_{kt}\omega_k A_k}\right)^{\frac{1}{\omega_k - 1}},$$

and substituting the solution for $\frac{l_{kt}h_t}{d_t}$ back into the production function (45),

$$\frac{\kappa_{kt}}{d_{kt}} = A_k \left(\frac{\omega_k A_k p_{kt}}{w_t} \right)^{\frac{\omega_k}{1-\omega_k}}. \tag{47}$$

Given that

$$\kappa_{kt} = (1 - a_{kt})\, s_{Gt} r_t k_t, \tag{48}$$

and that deposits d_{kt} equal capital income $s_{Gt} r_t k_t$, it follows that

$$1 - a_{kt} = A_k \left(\frac{p_{kt} \omega_k A_k}{w_t} \right)^{\frac{\omega_k}{1-\omega_k}}; \tag{49}$$

this is the same solution for $1 - a_{kt}$ as in equation (15) except that now w_t replaces r_t. Also, since the CRS banking production property implies that

$$1 - \omega_k = \frac{r_{kt} d_{kt}}{p_{kt}\,(1 - a_{kt})\, s_{Gt} r_t k_t}; \tag{50}$$

given that $d_{kt} = s_{Gt} r_t k_t$, it results as in equation (18) that $r_{kt} = p_{kt}\,(1 - \omega_k)\,(1 - a_{kt})$.

3.3 Government

Government revenue is equal to the lump sum transfer of v_t :

$$a_{kt} \tau_k r_t s_{Gt} k_t = v_t. \tag{51}$$

3.4 Equilibrium: Growth Effect of Tax

Proposition 1 again results, whereby $p_{kt} = \tau_k$. And the balanced-growth path equilibrium is again the same is in equation (22) and (26).[7] But now when the tax rate changes, there is an additional effect on the growth rate, as compared to that of Proposition 2, since the real interest rate now changes whereas before it was constant.

With the growth rate equal to $g_t = r_t\,(1 - \tau_k + r_{kt}) - \delta_K - \rho$, as in equation (22), the change in the growth rate from a tax rate change is comprised of two effects:

$$\frac{\partial g}{\partial \tau_k} = r_t \frac{\partial\,(1 - \tau_k + r_{kt})}{\partial \tau_k} + (1 - \tau_k + r_{kt})\,\frac{\partial r_t}{\partial \tau_k}.$$

[7] In equation (27), w_t replaces r_t in the solution for $1 - a_{kt}$.

The first term is as in equation (23), $r_t \frac{\partial(1-\tau_k+r_{kt})}{\partial\tau_k} = -r_t a_{kt}$, except that here r_t is not equal to the constant A_G. Substituting this in,

$$\frac{\partial g}{\partial \tau_k} = -r_t a_{kt} + (1 - \tau_k + r_{kt}) \frac{\partial r}{\partial \tau_k}. \tag{52}$$

The second term $(1 - \tau_k + r_{kt}) \frac{\partial r_t}{\partial \tau_k}$ is a secondary effect that is affected by human capital as given generally in Stokey and Rebelo (1995).

In this model, the returns to human and physical capital are equal along the balanced growth path, and in particular are given by

$$r_t (1 - \tau_k + r_{kt}) - \delta_K = (1 - \varepsilon) A_H \left(\frac{l_{Ht} h_t}{s_{Ht} k_t}\right)^{\varepsilon} (1 - x_t) - \delta_H. \tag{53}$$

The marginal product of effective labor in human capital production (the derivative of equation 36) is given by the $(1 - \varepsilon) A_H \left(\frac{s_{Ht} k_t}{l_{Ht} h_t}\right)^{\varepsilon - 1}$ part of equation (53). The other part of the human capital rate of return is the "capacity utilization rate" of human capital, or $1 - x_t$, which is the productive employed time that is factored by h_t.

The capital tax increase drives down the return to both physical and human capital. Factor realignment in the face of the tax, from more heavily taxed capital to untaxed effective labor, causes the capital to effective labor ratios to fall across both goods and human capital sectors, as the factor input ratio r_t/w_t rises. Employed time stays about the same, with less goods production using less labor time, but more human capital production using more human capital investment time. The upshot is an increase in r_t.

However, the effect on r_t is of secondary order in terms of magnitude, as compared to the first term of equation (52). And the fraction of reported income a_{kt} determines this effect, as in Proposition 2, and again a_{kt} is the determinate of changes in the price elasticity of demand for the reported income, since the elasticity equations (28) and (29) still hold in this economy.

Since r_t does increase as a result of the tax, the growth effect of the tax is somewhat less in this economy as compared to the Section 2 Ak economy. But the nonlinearity effect of the tax evasion is only slightly affected. If the productivity of the bank sector is set to zero, so that $A_k = 0$, $a_{kt} = 1$ and

$r_{kt} = 0$, then the growth effect is

$$\frac{\partial g}{\partial \tau_k} = -r_t + (1 - \tau_k) \frac{\partial r}{\partial \tau_k}, \tag{54}$$

and the economy returns to the standard tax effect analysis of Stokey and Rebelo (1996). Simulations give the same general shape of the capital tax - growth profile as in Figure 2; with $A_k > 0$ and some degree of tax evasion, the tax-growth profile is nonlinear with the marginal effect on growth smaller as the tax rate increases (see Section 4.4).

4 Extensions to other Taxes

Consider how tax evasion would affect the response of the growth rate to a labor tax increase. To focus on this, first an Ah economy with only human capital is presented, in which there is a tax only on labor. Then this labor tax is imposed along with the capital tax in the economy with human and physical capital. And finally in this latter setting, a goods tax, or value-added tax, is also imposed.

4.1 Labor Tax, Evasion, and Only Human Capital

Similar to the Ak economy of Section 2, the Ah economy has an analytic solution for the effect of the tax on the growth rate when tax evasion is produced in the banking sector. The consumer has the same utility as in the Section 3 economy, of equation (31). Time is allocated between the three sectors, similar to equation (32), but now with time also used to produce banking, l_{kt} :

$$l_{Gt} + l_{Ht} + l_{kt} + x_t = 1. \tag{55}$$

Goods production is linear in the effective time $l_{Gt}h_t$:

$$y_t = A_G l_{Gt} h_t. \tag{56}$$

And human capital investment is also linear in effective time, as in Lucas (1988), in a no-physical-capital modification of equation (36):

$$i_{Ht} = A_H l_{Ht} h_t. \tag{57}$$

The human capital accumulation equation (34) again applies.

The tax rate on labor income is denoted by τ_l, and the fraction of income that the consumer reports to the government is denoted by a_{lt}. The bank fee paid for the tax evasion service is p_{lt}, and the dividend rate received from depositing labor income in the tax evasion bank is denoted by r_{lt}. The quantity of deposits in the bank, denoted by d_{lt} is the labor income from goods and banking production, or $w_t (l_{Gt} + l_{lt}) h_t$. And for the fraction of income, $(1 - a_{lt}) w_t (l_{Gt} + l_{lt}) h_t$, upon which the consumer evades taxes, the total fees paid are $p_{lt} (1 - a_{lt}) w_t (l_{Gt} + l_{lt}) h_t$. With additional income of the government tax lump sum transfer v_t, and the expenditure on consumption of c_t, the consumer budget constraint, instead of equation (38) of the last section, is now:

$$(1 - \tau_l) a_{lt} w_t (l_{Gt} + l_{lt}) h_t + (1 - p_{lt}) (1 - a_{lt}) w_t (l_{Gt} + l_{lt}) h_t + r_{lt} d_{lt} + v_t - c_t = 0. \tag{58}$$

The additional constraint on the consumer problem, instead of equation (39), is that the deposits in the bank equal the labor income deposited:

$$d_{lt} = w_t (l_{Gt} + l_{lt}) h_t. \tag{59}$$

The goods producer problem, instead of equation (8), is to maximize profit of

$$\max_{\{l_{Gt} h_t\}} \Pi_{Gt} = A_G (l_{Gt} h_t) - w_t l_{Gt} h_t, \tag{60}$$

so that in equilibrium,

$$w_t = A_G. \tag{61}$$

The government budget constraint, similar to equation (19), is

$$a_{lt} \tau_l w_t (l_{Gt} + l_{lt}) h_t = v_t. \tag{62}$$

The bank supplying the tax evasion service, in a fashion similar to equation (44), maximizes profit

$$\max_{\{l_{lt} h_t\}} \Pi_{lt} = p_{lt} \kappa_{lt} - w_t l_{lt} h_t - r_{lt} d_{lt}; \tag{63}$$

subject to the production function:

$$\kappa_{lt} = A_l \left(l_{lt} h_t\right)^{\omega_l} \left(d_{lt}\right)^{1-\omega_l}, \tag{64}$$

with $\omega_l \in (0,1)$. And similar to equation (47), in equilibrium,

$$\frac{\kappa_{lt}}{d_{lt}} = A_l \left(\frac{\omega_l A_l p_{lt}}{w_t}\right)^{\frac{\omega_l}{1-\omega_l}}. \tag{65}$$

Solving for $1 - a_{lt,}$ similar to equation (15), it results that

$$1 - a_{lt} = A_l \left(\frac{p_{lt} \omega_l A_l}{w_t}\right)^{\frac{\omega_l}{1-\omega_l}}. \tag{66}$$

In equilibrium, the consumer maximizes utility subject to the human capital investment, income and deposit constraints, of equations (34), (57), (58) and (59). It results as in Proposition 1 that

$$p_{lt} = \tau_l. \tag{67}$$

The price of the tax evasion service is the labor tax rate, so that the supply of laundered income is equal to

$$1 - a_{lt} = A_l \left(\frac{\tau_l \omega_l A_l}{w_t}\right)^{\frac{\omega_l}{1-\omega_l}}, \tag{68}$$

which can be graphed as in Figure 1. The dividend rate is given by

$$r_{lt} = \tau_l \left(1 - \omega_l\right) \left(1 - a_{lt}\right);$$

and the growth rate is given by

$$g_t = A_H (1 - x_t) - \delta_H - \rho, \tag{69}$$

where $A_H \left(1 - x_t\right) - \delta_K$ is the return on human capital, a simplified return as compared to that in equation (53) because of the lack of physical capital.

Growth rate effects of the tax are less apparent in equation (69), compared to the Ak economy's equation (22). The labor tax acts on growth through its effect on leisure x_t. The marginal rate of substitution between goods and leisure, $MRS_{c,x}$, in this economy is

$$MRS_{c,x} = \frac{x_t}{\alpha c_t} = \frac{1}{(1 - \tau_l + r_{lt})\, w_t h_t}. \tag{70}$$

The effective tax rate is equal to $\tau_l - r_{lt}$, similar to Corollary 1. A tax increase lowers the shadow price of leisure and causes substitution from goods to leisure. The increase in leisure causes the growth rate to fall.

Substituting the equilibrium leisure into the growth rate equation (69), the result is that the growth rate falls by less as the tax rate increases, now that there is tax evasion; Figure 3 illustrates this.[8] The economy with no

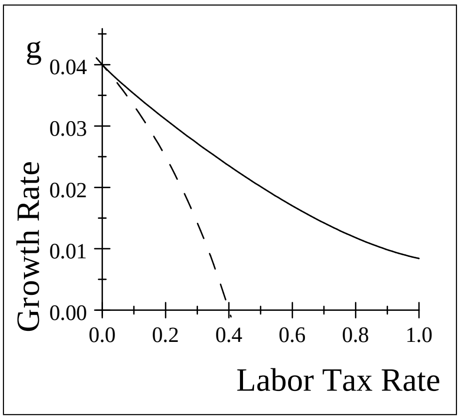

Figure 3: The Effect on Growth of the Labor Tax τ_l with Tax Evasion

tax evasion (dashed line) indicates that the growth rate falls at an increasing rate as the labor tax increases. With tax evasion, there is a diminished fall in the growth rate at all tax rate levels, with the growth rate decrease being of a diminishing magnitude at higher tax rates, as was found for the capital tax in Section 2.

The price elasticity of the demand for the reported income, per unit of human capital rises in magnitude as the tax rate rises. This can be seen by

[8] The solution for leisure is
$x_t = \frac{\alpha\rho\left[1 - A_G\left(\frac{\tau_l \omega_l A_l}{A_G}\right)^{\frac{1}{1-\omega_l}}\right]}{A_H\left[1 - \tau_l + (1-\omega_l)\tau_l A_l\left(\frac{\tau_l \omega_l A_l}{A_G}\right)^{\frac{\omega_l}{1-\omega_l}}\right]}$. With $A_l = 0$, and no tax evasion, then $x = \frac{\alpha\rho}{A_H(1-\tau_l)}$ and growth effect reduces to $\frac{\partial g_t}{\partial \tau_l} = -x_t A_H\left[\frac{1}{1-\tau_l}\right]$. Parameters assumed in Figure 3 are $A_H - \delta_K = 0.12$, $\rho = 0.02$, $\alpha = 3$, $\omega_l = 0.3$, $A_G = 0.19$, and $A_l = 0.7$.

taking the quantity of reported income, per unit of human capital, which is $w_t(l_{Gt}+l_{lt})a_{lt}$, defining the elasticity as $\eta_{\tau_l}^{w_t(l_{Gt}+l_{lt})a_{lt}} \equiv \frac{\frac{\partial[w_t(l_{Gt}+l_{lt})a_{lt}]}{\partial\tau_l}}{\frac{w_t(l_{Gt}+l_{lt})a_{lt}}{\tau_l}}$, and then using the fact that in this economy $l_{Gt}+l_{lt}=\frac{\rho}{A_H}$,[9] and $w_t=A_G$, to compute the elasticity as $\eta_{\tau_l}^{w_t(l_{Gt}+l_{lt})a_{lt}} = \eta_{\tau_l}^{a_{lt}} = -\frac{1-a_t}{a_{lt}}\frac{\omega_l}{1-\omega_l}$; this is similar to equation (28) of the Ak economy. By using the solution for a_{lt} in equation (68), this shows that the price elasticity of demand for the normalized reported income rises in magnitude as the tax rate increases, while at the same time the decrease in the growth rate becomes less.

4.2 Labor and Capital Taxes in the Full Economy

Placing both labor and capital taxes within the Section 3 economy, with both human and physical capital, shows that the principle of a rising price elasticity and a lesser growth rate decrease holds for both taxes simultaneously in this extended setting. The simplest way to present this economy is to allow for two separate banks, one that processes capital income and one that processes labor income. The Sections 3 and 4 assumptions are maintained that only labor and deposits are used both in the capital income processing bank and in the labor income processing bank.

With the same utility function, the consumer's budget constraint is now:

$$\begin{aligned}\dot{k}_t &= a_{lt}(1-\tau_l)w_t(l_{Gt}+l_{kt}+l_{lt})h_t+(1-a_{lt})(1-p_{lt})w_t(l_{Gt}+l_{kt}+l_{lt})h_t \\ &\quad +r_{lt}d_{lt}+a_{kt}(1-\tau_k)r_t s_{Gt}k_t+(1-a_{kt})(1-p_{kt})r_t s_{Gt}k_t+r_{kt}d_{kt} \\ &\quad -c_t+v_t-\delta_K k_t. \qquad (71)\end{aligned}$$

And the deposit constraints are that

$$d_{kt}=r_t s_{Gt}k_t;$$

$$d_{lt}=w_t(l_{Gt}+l_{kt}+l_{lt})h_t.$$

[9] This is found by dividing i_{Ht} in equation (57) by h_t, which equals g_t on the balanced-growth path, and setting this equal to g_t of equation (69).

The government budget constraint now is

$$a_{kt}\tau_k r_t s_{Gt} k_t + a_{lt}\tau_l w_t \left(l_{Gt} + l_{kt} + l_{lt}\right) h_t = v_t. \tag{72}$$

The same human capital technology as in Section 3 applies (equation 36). Then the growth rate equation is again the same as in equation (22), and the equality of the returns to physical and human capital is given by equation (53). And now a labor tax affects leisure, which affects growth mainly through the tax effect on leisure that determines the return to human capital in equation (53). The labor tax also has a secondary order effect on the real interest rate in the general equilibrium. Similarly the capital tax's primary effect is on the return to physical capital, with a secondary effect on leisure and the return to human capital.

Simulations for the effects of the capital and labor tax indicate the same shapes as in Figures 2 and 3: a more non-linear profile when there is tax evasion versus the more linear profile when $A_k = A_l = 0$ and there is no tax evasion (see Section 4.4).

4.3 Adding a VAT Tax

Finally, consider additionally adding a VAT goods tax and again a separate bank for the goods tax evasion. This is presented within the full economy with both physical and human capital, and with both capital and labor taxes and their evasions, as well, as in the last subsection. The results are a simple extension of those results already obtained for the capital and labor taxes.

To see this, let there be a proportional tax on the sales of goods purchases, denoted by τ_c, similar to a value-added tax (VAT). Then the cost of buying goods is now $(1 + \tau_c)\, c_t$ if all of the sales are reported to the government. Typically in the representative agent growth model without exchange we consider the goods to be bought and sold from the producer, rather than adding the layer of stores that sell the goods. Here, think of the consumer as owning the stores that distribute the goods. Then when there is a tax on sales, the consumer has to pay the tax when buying the good, and the store owner then has to report the total sales and receipts to the government. By buying tax evasion banking services, at a per dollar price of p_{ct}, the consumer is able to pay sales taxes on only a portion of the goods, and can evade taxes on the rest. This is similar to the practice of paying a lower price for some purchases because it is understood that the sale is "under-the-counter", will not be reported for tax accounting, and so will not include the tax in the price.

Let a_{ct} be the fraction of goods purchases that are reported to the government and $1 - a_{ct}$ the fraction that is unreported. The bank providing the goods tax evasion is owned by the consumer according to the amount of deposits made in the bank. The deposits, d_{ct}, are equal to the amount of consumption sales c_t :

$$d_{ct} = c_t; \tag{73}$$

and the consumer receives a dividend per deposit as denoted by r_{ct}. The quantity of the tax evasion services being demanded are given by $\kappa_{ct} = (1 - a_{ct}) c_t$. Therefore the total fee paid for the services is $p_{ct} (1 - a_{ct}) c_t$.

In producing the bank VAT evasion services, it is again assumed that only effective labor and deposits are inputs. The consumer spends additional labor time l_{ct} working for this new bank, and now instead of equation (71), the consumer budget constraint is:

$$\begin{aligned}\dot{k}_t &= a_{lt} (1 - \tau_l) w_t (l_{Gt} + l_{kt} + l_{lt} + l_{ct}) h_t + a_{kt} (1 - \tau_k) r_t s_{Gt} k_t \\ &\quad + (1 - a_{lt}) (1 - p_{lt}) w_t (l_{Gt} + l_{kt} + l_{lt} + l_{ct}) h_t \\ &\quad + (1 - a_{kt}) (1 - p_{kt}) r_t s_{Gt} k_t - (1 + \tau_c) a_{ct} c_t - (1 + p_{ct}) (1 - a_{ct}) c_t \\ &\quad + r_{lt} d_{lt} + r_{kt} d_{kt} + r_{ct} d_{ct} + v_t - \delta_K k_t. \end{aligned} \tag{74}$$

The consumer problem is as in the previous section except that now the budget constraint is equation (74), and there is an additional deposit constraint given by equation (73). The goods producer problem is the same, and the bank problems are the same for the banks providing the capital and labor income evasion services. There is an additional bank with the profit Π_{ct} given by

$$\max_{\{l_{ct} h_t\}} \Pi_{ct} = p_{ct} \kappa_{ct} - w_t l_{ct} h_t - r_{ct} d_{ct}; \tag{75}$$

subject to the production function:

$$\kappa_{ct} = A_c \left(l_{ct} h_t\right)^{\omega_c} \left(d_{ct}\right)^{1-\omega_c}, \tag{76}$$

with $\omega_c \in (0,1)$. In equilibrium,

$$\left(1 - a_{ct}\right) = A_c \left(\frac{\omega_c A_c p_{ct}}{w_t}\right)^{\frac{\omega_c}{1-\omega_c}}. \tag{77}$$

And the government budget constraint becomes

$$a_{kt} \tau_k r_t s_{Gt} k_t + a_{lt} \tau_l w_t \left(l_{Gt} + l_{kt} + l_{lt} + l_{ct}\right) h_t + \tau_c a_{ct} c_t = v_t. \tag{78}$$

The consumer equilibrium conditions imply the equivalence between the VAT rate and the price of evasion services:

$$p_{ct} = \tau_c;$$

and the dividend return is

$$r_{ct} = \tau_{ct} \left(1 - \omega_c\right) \left(1 - a_{ct}\right). \tag{79}$$

The marginal rate of substitution between goods and leisure becomes

$$MRS_{c,x} = \frac{x}{\alpha c} = \frac{1 + \tau_c - r_{ct}}{\left(1 - \tau_l + r_{lt}\right) wh}, \tag{80}$$

which differs from equation (70) by the addition of the effective VAT tax rate in the numerator of $\tau_c - r_{ct}$. The equality of returns of human and physical capital along the balanced-growth path is again given by equation (53), and the growth rate by equation (22).

The effect of the VAT of course is to reinforce the goods to leisure substitution that the labor tax also induces. Thus the growth effect of the tax again works mainly through its effect on the amount of leisure taken, just as with the labor tax. Simulations indicate that the growth rate falls at a decreasing rate as the VAT rises, and that there is a less nonlinear profile when $A_c = 0$ and there is no tax evasion service for the VAT (see the next subsection).

4.4 Simulations

This section shows the simulated profile of the tax rate on the growth rate in each of four cases. Besides the three taxes on capital income, labor income, and the goods VAT tax, as described in this section, a monetary extension is made whereby the inflation tax is added and avoided through an additional bank sector that produces credit with the same technology as that used to evade the other taxes. This credit economy is found in Benk, Gillman, and Kejak (2008).[10]

An illustrative calibration sets standard values for parameters: the share of capital in the goods and human capital sectors, $\beta = \varepsilon = 0.36$, physical and human capital depreciation rates, $\delta_K = \delta_H = 0.05$, the discount rate, $\rho = 0.04$ and log-utility. Given a growth rate of the economy of $g = 0.02$, the weight of leisure in utility function set at $\alpha = 1.9$, and with productivity parameters of $A_G = 1.5$, and $A_H = 0.233$, leisure is $x = 0.6$.

The labor shares in the bank sectors for evading the capital, labor and VAT taxes are $\omega_k = \omega_l = \omega_c = 0.3$ and the productivity parameters are assumed to be $A_k = A_l = A_c = 1$. For inflation tax avoidance through credit use, the comparable "omega" labor share in credit production, call it ω_D, is set at 0.2, and the comparable bank productivity factor is set at $A_D = 0.77$. The tax rates, when they are not varying in the simulation, are set equal to $\tau_k = \tau_l = \tau_c = 0.15$. The money growth rate, σ, is set at 0.07, when not varying, giving an inflation rate of $\pi = 0.05$; variation in σ is used in Figure 4 (panel a) to show the effect of the inflation tax on growth.

Figure 4 shows the tax-growth profile for a range of tax rates for each of the four taxes (panels a,b,c, and d), when there is tax evasion or avoidance. The dashed line shows how the profile shifts down as the bank productivity goes down (by 10% of its initial value). When increasing any of the given taxes, the decrease in the growth rate is smaller (solid lines) than when there is no tax evasion (not shown) or the tax evasion is less productive (dashed lines).

[10] In Benk et al (2008), there are no labor or capital taxes, and there are stochastic shocks.

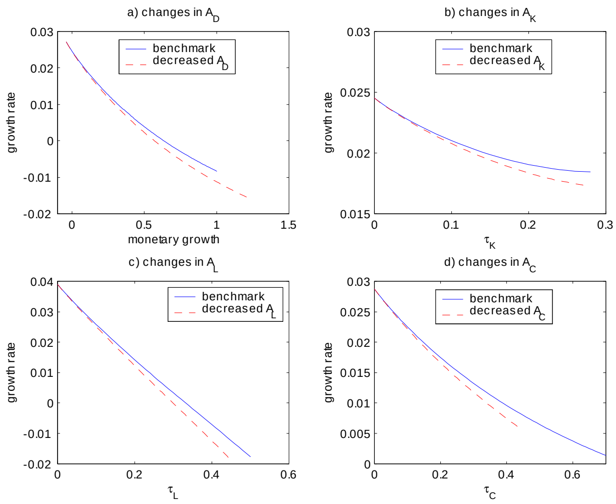

Figure 4: Growth Effect Across Different Taxes; Bank Productivity Decrease

5 Conclusion

The paper postulates a principle of public finance for the effect of taxes on endogenous growth when tax evasion is allowed. This is developed most simply within an Ak economy, with only a capital income tax, and then extended to labor income and goods sales taxes. And the results are parallel to those in Gillman and Kejak (2005) for the inflation tax. The tax evasion allows for the effective tax rate to be decreased. As the tax rises, the elasticity of substitution between reporting the income and not reporting the income rises; and the price elasticity of demand for the reported income with respect to the tax also increases in magnitude. This substitution away from reporting the income (or sales) causes the effective tax rate to fall as the tax rate rises. As a result, tax evasion causes the growth rate to fall by marginally less for marginally higher tax rates, as compared to no tax evasion.

The model developed to generate these results does not rely on preferences for evasion. Rather a competitive banking sector is specified that produces the tax evasion service at a competitively determined price. In equilibrium, this proportional tax evasion fee equals the tax rate, a traditional result whereby the consumer is willing to spend on the margin an amount equal to the cost of the tax. The result follows from using a micro-founded production function for the banking sector as found in the financial intermediation services literature starting with Sealey and Lindley (1977), Clark (1984) and Hancock (1985), and now established as the dominant approach in the banking.[11] While convex cost functions are sometimes assumed to establish the banking service equilibrium (Berk and Green 2004), here the general equilibrium incorporation of the intermediation approach gives such an upward sloping marginal cost of evasion per dollar unit, as in Figure 1, and provides the basis of the paper's results.[12]

The tax evasion model here extends the famous Baumol (1952) exchange trade-off, whereby the marginal cost of avoiding money use through banking is equal to the nominal interest rate (which is the inflation tax rate given an optimum of a zero nominal interest rate). Adding tax evasion in a non-monetary context, with the marginal cost of evasion equal to the tax rate, results in an intuitively plausible nonlinear tax-growth profile for each tax. And the reasons for this are plausible as well, a rising price elasticity to the taxed good as the tax rate rises. However, while such a tax-growth profile has empirical support with respect to the inflation tax, the empirics of the tax-growth profiles remains to be investigated for the capital, labor and goods taxes, a topic for future research.

A major qualification is that the analysis is a positive one about growth rate effects, with interesting normative questions, on the optimal structure of taxes in this environment, left for future research. Conditions can be stated by which welfare is lower, in the Section 2 Ak and Section 4.1 Ah economies, given that there is tax evasion ($A_k > 0$; $A_l > 0$) and there are low tax rates, even though the growth rate is higher; but welfare can also be higher with tax evasion at sufficiently high taxes. And examining welfare in the full economy with human and physical capital also requires consideration of transitional dynamics, when the tax rate changes. Second-best Ramsey considerations also remain for future work.

[11] According to Matthews and Thompson (2008); see for example Berger and Mester (1997).

[12] If deposits are not used as an input, and only labor or capital are inputs, then no unique equilibrium exists; see Proposition 1, Gillman, Harris, and Kejak (2007).

Other qualifications are clear: tax evasion causes less tax collection and so can force greater reliance on less efficient taxes that result in lower growth in the end. And evasion activity induces an income loss from the using up of real resources, the amount of which depends on the productivity of banking. And bank verification and asymmetric information issues are here captured abstractly with the productivity parameter in the bank production. But abstracting from informational issues and the complications of the optimal amount of enforcement to keep evasion under control, itself a subject for full inquiry (Ehrlich 1996, Becker, Murphy, and Grossman 2006), the point here is that we can see formally how competitive evasion activity, with a lump sum return of government revenue, lowers the effective tax rate, increases the price elasticity of demand for the declared income, and increases growth. And with evasion, a decrease in the tax rate results in a bigger marginal increase in growth, the lower is the initial tax rate.

References

Baumol, W. (1952): "The Transactions Demand for Cash: An Inventory -Theoretic Approach," *Quarterly Journal of Economics*, 66, 545–566.

Becker, G. S. (1965): "A Theory of the Allocation of Time," *Economic Journal*, 75(299), 493–517.

Becker, G. S. (1968): "Crime and Punishment: An Economic Approach," *The Journal of Political Economy*, 76(2), 169–217.

Becker, G. S., K. M. Murphy, and M. Grossman (2006): "The Market for Illegal Goods: The Case of Drugs," *Journal of Political Economy*, 114(1), 38–60.

Benk, S., M. Gillman, and M. Kejak (2008): "Money Velocity in an Endogenous Growth Business Cycle with Credit Shocks," *Journal of Money, Credit, and Banking*, Forthcoming.

Berger, A. N., and L. Mester (1997): "Inside the Black Box: What Explains Differences in Efficiencies of Financial Institutions," *Journal of Banking and Finance*, 21, 895–947.

Berk, J., and R. Green (2004): "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112(6), 1269–1295.

Clark, J. A. (1984): "Estimation of Economies of Scale in Banking Using a Generalized Functional Form," *Journal of Money, Credit, and Banking*, 16(1), 53–68.

Dougherty, C. (2008): "U.S. Among Countries Investigating Tax Evasion," *New York Times*, p. www.nytimes.com.

Ehrlich, I. (1996): "Crime, Punishment, and the Market for Offenses," *The Journal of Economic Perspectives*, 10(1), 43–67.

Gillman, M., M. Harris, and L. Matyas (2004): "Inflation and Growth: Explaining a Negative Effect," *Empirical Economics*, 29(1), 149–167.

Gillman, M., M. N. Harris, and M. Kejak (2007): "The Interaction of Inflation and Financial Development with Endogenous Growth," Discussion paper, Money Macro and Finance Research Group Conference 2006.

Gillman, M., and M. Kejak (2005): "Inflation and Balanced-Path Growth with Alternative Payment Mechanisms," *Economic Journal*, 115(500), 247–270.

Hancock, D. (1985): "The Financial Firm: Production with Monetary and Nonmonetary Goods," *Journal of Political Economy*, 93(5), 859–880.

King, R. G., and S. Rebelo (1990): "Public Policy and Economic Growth: Deriving Neoclassical Implications," *Journal of Political Economy*, 98(2), S126–S150.

Lucas, Jr., R. E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.

MARSHALL, A. (1920): *Principles of Economics*. The Macmillan Press Ltd, London, eighth edn.

MATTHEWS, K., AND J. THOMPSON (2008): *The Economics of Banking*. John Wiley and Sons, Ltd, West Sussex, second edn.

MOLLENKAMP, C., G. R. SIMPSON, AND A. FRANGOS (2008): "Two Indicted in UBS Case: U.S. Officials Say Former Banker Helped Client Evade Taxes," *Wall Street Journal*, p. 17.

SCHNEIDER, F., AND D. ENSTE (2000): "Shadow Economies: Size, Causes, and Consequences," *Journal of Economic Literature*, 38(1), 77–114.

SEALEY, C. W., AND J. T. LINDLEY (1977): "Inputs, Outputs, and a Theory of Production and Cost at Depository Financial Institutions," *Journal of Finance*, 32(4), 1251–1266.

STOKEY, N. L., AND S. REBELO (1995): "Growth Effects of Flat-Rate Taxes," *Journal of Political Economy*, 103(3), 519–550.

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